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Interaction between Rossby Waves and a Jet Flow: Basic Equations and Verification for the Antarctic Circumpolar Current

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Abstract—The article focuses on the interaction of Rossby waves in the ocean with zonal jet flows. A new approach is proposed to show that nonlinearity in the long-wave approximation exactly compensates the Doppler shift. A new dispersion relation for the Rossby waves interacting with the jets is deduced from the nonlinear theory. The conclusion is verified using satellite altimetry data of the Antarctic Circumpolar Current (ACC). For the ACC area, we compare empirical velocities obtained from the altimetry data with theoretical phase velocities of Rossby waves calculated from nonlinear dispersion relation using the equivalent beta effect. The comparison shows that the new dispersion relation based on the nonlinear approach is capable of describing both the westward and the eastward propagation of mesoscale eddies in the field of sea level anomalies that can be identified as manifestation of Rossby waves in the ocean.

Keywords: Rossby waves, mesoscale eddies, jet current, Antarctic Circumpolar Current, dispersion relation, nonlinear theory, altimetry

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INTRODUCTION

In recent years, the progress observed in the field of remote sensing of the Earth has contributed to the development of empirical ideas about Rossby waves in the ocean. As indicated by Nezlin [1], Rossby waves in the ocean appear as mesoscale eddies and can now be studied in different parts of the oceans by analyzing, for example, satellite altimetry data [2–6]. The speeds of mesoscale eddy propagation, in general, are in good agreement with the dispersion relations for the baroclinic Rossby waves, although many authors note that the empirical velocities somewhat exceed the theoretical phase velocities obtained in the linear approximation for harmonic waves [4–10].

Nevertheless, the linear theory is not valid in the region of powerful jet flows. In particular, the property of the Rossby waves propagating to the west is violated. According to the satellite observations, in the region of the Antarctic Circumpolar Current (ACC), mesoscale eddies move not only westward, but also eastward [4, 11, 12]. Killworth et al. [13] associate this fact with the Doppler shift of the mean velocity of the ACC propagating eastward, noting that there are no strict estimates of the mean flow velocity. However, when Rossby waves interact with the flow, a significant transformation of the parameters of the waves, as well as the frequency and phase velocity of the waves change caused by the Doppler effect due to their interaction with the flow.

The ACC is the most powerful current in the World Ocean. It is located approximately between 40° and 65° S, flowing around the Antarctic continent from west to east. The transport of the ACC is 144 Sv (1 Sv = 10^6 m^3 s^{-1}). According to modern concepts, the ACC extends to a considerable depth and has an equivalent barotropic structure, i.e., the streamlines in the deep layers are parallel to the streamlines on the surface [14]. Tarakanov [15] in his study notes that today there are no unambiguous ideas about the structure of the ACC and its jets, although their correct description is essential for a correct assessment of the eddy transport in the ACC, which is one of the three components of the balance of the global ocean conveyor in the Southern Ocean, along with the flow of purely drift current and deep geostrophic currents. The Southern Ocean as a whole and the ACC play a key role in the transfer of wind forcing to the entire belt of the Global Ocean Conveyor.

It was shown in papers [11, 12, 16], based on a comprehensive analysis of altimetry measurements and modeling data, that the ACC is the waveguide of Rossby waves in the ocean. Here, the energy of Rossby waves is trapped and their structure is transformed: the waves are blocked by a powerful stream and their constant propagation in the western direction stops. In the ACC region, the Rossby waves and mesoscale eddies can propagate both to the west and to the east. The authors of the paper [16] indicate that phase velocities of wave propagation in the eastern direction in the ACC are 1–3 cm/s, while emphasizing that these velocities are much lower: one or two orders lower than the velocity of the typical surface currents in the ACC core. This fact does not allow us to use the dispersion relations while considering the interaction of waves with the flow as a simple Doppler shift, as suggested in the work [11], since the strong difference in the velocities of waves and currents makes this theory in general inapplicable to describing the processes under consideration. At the same time, Hughes [12] and later other authors showed that nonlinear effects dominate in the interaction of the Rossby waves with a strong flow [17, 18].

The problem of the waveguide and the formation of the critical layer for the barotropic and baroclinic Rossby waves in the zonal current in the atmosphere and ocean was analyzed in the works [19–21]. In these studies, the meridional length of the zonal flows was assumed to be substantially greater than the Rossby wavelength, and the latitude at which the meridional component of the wavenumber was zero (reflection condition) was considered the waveguide boundary. Since the phase velocities of low-frequency wave disturbances, as the authors considered, have the same order as the velocities of the background currents, critical layers play a special role, where these velocities coincide. It turned out that the nonlinear interaction of the barotropic Rossby waves and flows leads to the reflection of these waves from the critical layer and a linear theory of dynamic processes in the vicinity of such layers does not work. In the work of Belonenko and Frolova [22], the geographical application of this approach is given, where the waveguide boundary is defined as an area, in which the phase velocity of Rossby waves becomes zero. In this work, the phase velocities of the waves were determined from the Hovmoller diagrams of sea level changes, constructed from satellite altimetry. Thus, the study of the dynamics of the Rossby waves in the vicinity of the critical layers in zonal flows acquires great importance.

Hochet et al. [23], using a 2.5-layer model, investigated large-scale baroclinic instability as a potential source of Rossby waves and large-scale variability in the ocean. They showed that this baroclinic instability appears in the mean flow when the phase velocities of the two vertical modes of the model are equal to the mean flow velocity. These unstable areas exist in each oceanic basin, especially in the regions of intensification of the western boundary currents, as well as in the ACC. The authors make an assumption that the unstable modes considered in the model should probably also be observed in the satellite altimetry data, but they do not compare this theory with the empirical data, probably because a theory based on the assumption that the wave and flow velocities are not equal does not allow one to give an adequate interpretation of the observations.

Killworth and Blundell [24, 25], using the WKBJ approximation for low frequencies compared to the inertial frequency, considered the eigenvalue problem for the dispersion relation of the Rossby waves in the presence of the mean flow, taking into account the gradients of large-scale topographic inhomogeneities of the ocean floor. They demonstrated a rich variety of wave behavior in the World Ocean, including trapping of solutions at specific latitudes and over large-scale topographic features of the bottom. However, the dispersion relation, which makes it possible to explain the interaction of the waves with the current with a significant difference in magnitudes of speed, has not been given by the authors.

Hughes [12] analyzed estimates of advection of relative and planetary vorticity. The author showed their equilibrium in the ACC region, indicating that the meridional gradient of the shear flow and the meridional gradient of planetary vorticity provide a comparable contribution to the equations of conservation of potential vorticity. However, even in this work, there is no theory describing the nonlinear interaction of the Rossby waves with the ACC, which is consistent with the observations.

The main problem of describing the interaction of Rossby waves with the ACC is that the suggested theories proceed from the assumption that the velocities of the mean flow and the waves are comparable, and the critical layer is formed only when the velocities become equal. However, a real possibility of estimating these rates appeared with the development of satellite altimetry and data accumulation. It turned out that the ACC speeds significantly exceed the speeds of the Rossby waves, characteristic of these latitudes. This fact makes the previously suggested theories describing Rossby waves in a current, in which the flow and wave velocities were assumed to be comparable or equal in magnitude, inapplicable.

In this paper, we suggest a new approach that is free from such restrictions. Within the framework of the nonlinear theory, we show that nonlinearity in the longwave approximation exactly compensates the Doppler shift, and this allows us to obtain a new dispersion relation for the Rossby waves in a jet stream. We test this dispersion equation by analyzing satellite altimetry in different regions of the ACC. The suggested nonlinear theory can be applied to other areas of the World ocean.

THE DATA

We use data of the Absolute Dynamic Topography (ADT) to measure the level of the ocean relative to the geoid, provided by AVISO+ (http://www.aviso.altimetry.fr/en/data/products/auxiliary-products/mss.html). The spatial resolution of altimetry data is 0.25° by latitude and longitude, and the time resolution is 7 days. The data for the period 1993–2016 are arrays combined from the data of all altimetry missions, including the Cryosat, TOPEX/Poseidon, Envisat, Jason, and ERS 1/2 satellites. The data are transformed to the nodes of a regular grid with a spatial resolution of 0.25° × 0.25°.

BASIC EQUATIONS

We consider a nonlinear equation for a quasi-geostrophic potential vortex for the ocean motions on a synoptic scale (see Pedlosky, Vol. 2, formula (6.8.11)):

$$\left[\frac{\partial}{\partial t}+\Psi'_x\partial_y-\Psi'_y\partial_x\right]\left[\nabla_h^2\Psi+\left(\frac{1}{S}\Psi'_z\right)'_z+\beta y\right]=0,$$

where Ψ is the streamfunction, $S=\frac{N^2}{f^2}$, N is the Brunt-Väisälä frequency, $\beta=\frac{df}{dy}$, and f is the Coriolis parameter. The equation for Ψ can be written using absolute vorticity $\zeta_0=V'_x-U'_y$ and the geostrophic velocities can be written using the streamfunction $\Psi=P$, where P is atmospheric pressure:

$$U=-P'_y,$$

$$V=P'_x,$$

U and V are the zonal and meridional velocity components.

Then,

$$\zeta_0=V'_x-U'_y=P''_{xx}+P''_{yy}=\nabla_h^2P=\nabla_h^2\Psi.$$

Eventually, we get

$$\left[\frac{\partial}{\partial t}+\Psi'_x\partial_y-\Psi'_y\partial_x\right]\left[\zeta_0+\left(\frac{1}{S}\Psi'_z\right)'_z+\beta y\right]=0. \quad (1)$$

Equation (1) is a complete nonlinear equation for the stream function.

In almost all studies on the interaction of Rossby waves with the large-scale flows, infinitesimal linear Rossby waves are considered at the first stage. Further, due to the barotropic-baroclinic instability, linear waves grow to finite magnitudes and the previously rejected nonlinear terms become important. Following this approach, we will linearize Eq. (1) by expansion with respect to small parameter ε.

We assume that the background current $\vec{U}=(U,V)$ is homogeneous by the vertical coordinate and it does not depend on the vertical coordinate z: $\vec{U}=\vec{U}(x,y)$

$$\Psi=\Psi_0(x,y)+\varepsilon\varphi_1(x,y,z,t),\quad \varepsilon\ll 1. \quad (2)$$

The first term in Eq. (2) is the background stationary current, and

$U(x,y)=-(\Psi_0(y))'_y$ is the zonal part of the background stationary current;

$V(x,y)=(\Psi_0)'_x$ is its meridional part.

Then, the following relation is true:

$$-(\Psi_0(y))'_y=U(y)=U,$$

$$-(\Psi_0(y))''_{yy}=U'_y.$$

The second term in Eq. (2) describes linear waves (2). We shall seek φ_1 in the form of harmonic waves:

$$\varphi_1=\varphi_1(z)\exp(i(kx+ly-\omega t)).$$

If the background current is strictly zonal, then it is an exact solution of the nonlinear vorticity Eq. (1). If the background current is not zonal, then an external factor must be added to the right-hand side of Eq. (1) that supports this current. This paper does not specify exactly the nature of this external factor. It is assumed that the topography explicitly affects only the background current and the topography is not explicitly described in the equations for perturbations. However, the influence of external factors may indirectly affect the disturbances through the relative vorticity of the background current.

We substitute (2) into (1) and divide the variables for the linear terms. Then, we get the following dispersion relation:

$$\begin{aligned}&(\omega-kU-lV)\left[k^2+l^2+\lambda\right]\\&+k(\beta+(\zeta_0)'_y)-l(\zeta_0)'_x=0,\end{aligned} \quad (3)$$

where $\lambda\varphi_1(z)=-\left(\frac{1}{S}\varphi'_{1z}\right)'_z$ are the eigen values of the vertical problem, which can be found, for example, using the WKB approximation. Then, the dispersion relation for Eq. (1) is written in the following form:

$$\omega=\frac{-k(\beta+(\zeta_0)'_y)+l(\zeta_0)'_x}{k^2+l^2+\lambda}+kU+lV. \quad (4)$$

Parameter λ may be determined as follows:

(a) if the Brunt–Väisälä frequency is constant, $N^2(z) = N_0 = \text{const}$, then for the upper boundary condition at the free surface (see [30], equation (18.6)) we get $\lambda = \lambda_0 = \dfrac{f^2}{gH} \equiv F$ for the barotropic mode, where the Coriolis parameter is $f = 2\Omega \sin\varphi$, Ω is the angular velocity of the Earth's rotation, g is the acceleration due to gravity, and H is the ocean depth; the barotropic mode corresponds to the zero eigen value for the upper boundary condition in the form of a "rigid lid" (see Pedlosky, 1984, formula (6.12.10)):

$\lambda = \dfrac{f^2\pi^2 m^2}{(N_0 H)^2}$ for the baroclinic mode, $m = 1, 2, 3...$, and the baroclinic Rossby deformation radius (for the first mode) is $L_R = \dfrac{N_0 H}{\pi f}$. The baroclinic modes do not "feel" the upper boundary condition in the sense that they exist both under the free surface and under the rigid lid.

(b) if the Brunt–Väisälä frequency is $N^2(z) \neq \text{const}$, then we apply the WKB approximation (Chelton et al., 1998) and get

$$L_R = \frac{\int_{-H}^{0} N(z)dz}{\pi f}.$$

The expression for phase velocity is written as

$$c = \frac{\omega}{k} = \frac{-(\beta + (\zeta_0)'_y) + \frac{l}{k}(\zeta_0)'_x}{k^2 + l^2 + L_R^{-2}} + U + \frac{l}{k}V. \quad (5)$$

However, as satellite altimetry shows, this dispersion relation is not applicable for describing Rossby waves in the region of jet streams, including the ACC; therefore, other approaches are required for compatibility of theory and practice. In the search of a mathematical model describing empirical results, one should move on to other theories.

The eigenvalues given are related to the problem with a horizontal bottom. In the case of topography, the bottom relief condition will enter the equation for the lower boundary condition [24, 25, 27, 28]. Due to the limited publication volume, the effects caused by bottom topography are not included in this work and are the subject of a separate discussion.

In this paper, we put forward a working hypothesis that, for an adequate description of the Rossby waves in a zonal flow, one should apply the model of long-wave asymptotics of nonlinear Rossby waves. This model allows us to explain the behavior of Rossby waves in the ACC.

The specific property of Rossby waves is that the monochromatic Rossby wave is simultaneously a solution of both the linearized and the complete nonlinear vorticity equation. At the same time, in the absence of currents, the dispersion relations of linear and nonlinear Rossby waves coincide and have the following form:

$$\omega_0 = \frac{-\beta k}{k^2 + l^2 + F}, \quad (6)$$

where ω_0 is the frequency of waves without taking the currents into account.

However, as is shown above (see (5)) in the presence of a barotropic zonal flow, the dispersion relation of linear Rossby waves has a simple Doppler shift, but instead of parameter β we consider $\beta^* = \beta - U''_{yy}$:

$$\omega = \omega_0 + kU = \frac{-\beta^* k}{k^2 + l^2 + F} + kU, \quad (7)$$

where the first term is the eigen frequency of linear Rossby waves and the second is the Doppler shift. For nonlinear Rossby waves, the situation is different.

For simplicity, we first consider the two-dimensional problem using a simple example of shallow water (the barotropic case). The vorticity equation is written as

$$\left[\frac{\partial}{\partial t} + \Psi'_x\partial_y - \Psi'_y\partial_x\right]\left[\nabla_h^2\Psi - F\Psi + \beta y\right] = 0. \quad (8)$$

We seek the solution in the following form:

$$\Psi = \Psi(y) + \varphi(x, y, t). \quad (9)$$

Note that, unlike (2), there is no longer any assumption about the smallness of parameter ε.

Substituting (9) into (8), we get

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\left[\nabla_n^2\varphi - F\varphi\right] + \varphi'_x\left[\beta + \frac{\partial}{\partial y}\left(\Psi''_{yy} - F\Psi\right)\right] + A = 0, \quad (10)$$

where

$$A \equiv J\left(\varphi, \nabla_n^2\varphi - F\varphi\right), \quad (11)$$

and J is the Jacobian.

For the solutions in the form of a plane wave, we have

$$\varphi_0 = \varphi\exp\left(i(kx + ly - \omega t)\right), \quad A = 0, \quad (12)$$

where k and l are the zonal and meridional wavenumbers.

We finally get

$$i\varphi(\omega - kU)(K^2 + F) + i\varphi k(\beta - U''_{yy} + FU) = 0, \quad (13)$$

where $K^2 = k^2 + l^2$.

From this we obtain the following dispersion relation for nonlinear Rossby waves in a zonal flow, assuming that the wave propagates almost zonally:

$$\omega = \frac{-k(\beta - U''_{yy} + FU)}{K^2 + F} + kU. \quad (14)$$

Let us write this equation in the following form:

$$\omega = \frac{-k(\beta - U''_{yy})}{K^2 + F} + kU - \frac{kFU}{K^2 + F}. \quad (15)$$

The first term in formula (15) is the eigen frequency of nonlinear Rossby waves, the second is the Doppler shift, and the third is the effect of nonlinearity.

One can see from relation (15) that, in the longwave approximation $K^2 \to 0$. The last two terms in (15) mutually compensate each other and nonlinear long Rossby waves no longer "feel" the flow. Long nonlinear Rossby waves no longer "feel" the absolute value of the background current velocity, while the dependence on the relative vorticity of the background current remains in the first term.

Thus, in the longwave approximation, that is, when, $K^2 \ll F$, we obtain the dispersion relation for nonlinear Rossby waves:

$$\omega = \frac{-k(\beta - U''_{yy})}{F} \quad (16)$$

or $\omega = \frac{-k\beta^*}{F}$, where $\beta^* = \beta - U''_{yy}$, β^* is called the equivalent β effect. Then we can write the following for the phase velocity of nonlinear Rossby waves in the longwave approximation:

$$c^* = -\beta^* \cdot R^2. \quad (17)$$

It is worth noting that for the first time we are introducing the following term to the dispersion relation for the Rossby waves U''_{yy} so that the meridional gradients of planetary vorticity, the β effect, and relative vorticity: component U''_{yy} simultaneously participate as terms of equations in formulas (16) and (5). This means that the Rossby waves transform their energy during their interaction with the flow according to the properties of the flow.

Thus, in this work, the following working hypothesis has been put forward. Waves should be considered nonlinear in the longwave approximation in the ACC region. It follows from the shallow-water theory that nonlinearity in the longwave approximation exactly compensates for the Doppler shift. Note that this is a fact that has not been previously noted by anyone, and this result is fundamentally new. It is considered that, in the framework of the KdV theory (the Korteweg de Vries equation), nonlinearity exactly compensates for dispersion spreading, which leads to the solutions in the form of solitary solitons. For the Rossby waves, the KdV relevance follows from the balance between nonlinearity and the Doppler shift, which actually would be more correct. Consequently, the relevance of the KdV to the description of the Rossby waves in the vicinity of the ACC acquires new grounds. Moreover, the transition from linear to nonlinear waves allows us not only to justify the disappearance of the Doppler shift in the dispersion relation, but also to remove the questions about the barotropic–baroclinic instability.

We note that the mechanism for the formation of nonlinear longwave solutions is described in [29]. The authors show that at large β initially linearly unstable modes are able to transfer energy to the longwave limit through the inverse cascade and stable nonlinear formations of finite amplitude are formed there. These formations do not lead to the fatal destruction of the background flow, but in some way coexist. They call this mechanism of formation of nonlinear finite-amplitude longwave formations "multistage instability."

Note that, since the pioneering works of Chelton [2, 4], all analytical and numerical works sought to solve the main problem: the discrepancy between empirical and theoretical estimates of the speed of Rossby waves, which sometimes differ by a factor of two (this theory resulted in significantly lower phase velocities than found from the analysis of altimetry maps). The main factors that were used to resolve the discrepancies were the topography and baroclinicity of the flow. Note that there is no consensus on this question. Kilworth [24, 25], unlike Chelton, came to the conclusion that topography gives a zero integral effect on wave disturbances.

At later stages, the manifestation of modulation instability is quite possible, which will lead to the appearance of separate nonlinear formations. Consequently, we will come to a logical conclusion about the transition to the KdV equation. In this problem, the nonlinearity compensates not the dispersion, which already tends to zero, but the Doppler shift.

The transition from the barotropic problem (shallow-water two-dimensional case) to the more general barotropic–baroclinic case (three-dimensional task) should be started with two- and three-layer models, but their analysis is beyond the scope of this work due to its limitations. A consideration of the general baroclinic case should begin with the model problems. For the exact transfer of the dispersion relation (17) to the baroclinic case, it can be assumed that the flow and the wave perturbation in the nonlinear case have a common vertical structure. Vertical modes do not seem to interact between each other, which allows nonlinear disturbances to slip along the background current.

The last phrase needs to be clarified. Let us consider the equation for thermal wind (formula (44.10) of the monograph by Le Blond and Mysak, vol. 2). It shows that the vertical structure (dependence on the z of the velocity field and, consequently, the dependence of the stream function) is uniquely related to the inclination of the isopycnals. Hence, if the waves had a vertical structure different from the vertical structure of the flow, then the propagation of the eddies through the flow would be accompanied by the encounter with two slopes of isopycnals, which would have serious consequences. This is exactly what happens in the linear problem of the baroclinic instability. The potential energy reserve, which is caused by the inclination of isopycnals, is released and turns into kinetic energy. Therefore, the coexistence of two solutions with different vertical structures at a given point at the same time is unlikely. Since the waves exist for quite a long time, then, using the terminology of Gill [31], one can say that the geostrophic adjustment of the two solutions occurs: waves and currents. This is possible only if their vertical structure is identical. Of course, most likely, they are not completely identical, but in the first approximation this can be ignored.

Nonlinearity is quite a complex concept. Following Nezlin [1], nonlinearity creates some additional gradients of the free surface. However, just in order to simply separate the variables and obtain a mathematical solution, it is possible to make an assumption that the vertical structures are identical, which subsequently allows us to obtain a mathematical solution. Actually, this assumption is quite logical and physically substantial, since otherwise the waves would have spent their energy for moving along the current and quickly decay, although it is possible that this happens in the regions where the flow changes its structure of isopycnals: the eddies are absorbed by the flow. However, proceeding from what follows, it is more likely that, due to the weak difference in the vertical structures, the eddies simply gradually spend energy as they pass through the isopycnals of the current and gradually decay.

Let us consider the baroclinic case.

Let us return to Eq. (1). The vorticity equation has the following form:

$$\left[\frac{\partial}{\partial t} + \Psi'_x \partial_y - \Psi'_y \partial_x\right]\left[\nabla_h^2 \Psi + \left(\frac{1}{S}\Psi'_z\right)'_z + \beta y\right] = 0. \quad (18)$$

In this case, we seek for the solution in the following form:

$$\Psi(x, y, z, t) = [\Psi_0(y) + \varphi(x, y, t)]Z(z), \quad (19)$$

where, $\varphi(x, y, t) = \exp(i(kx + ly - \omega t))$.

We solve the vertical eigenvalue problem

$$-\lambda Z(z) = \left(\frac{1}{S} Z(z)'_z\right)'_z,$$

and get the following dispersion relation:

$$\omega = \frac{-k(\beta - U''_{yy} + \lambda U)}{K^2 + \lambda} + kU, \quad (20)$$

which is written as (17) if $K^2 \ll F$.

For a more general baroclinic case in the nonlinear case, the question arises as to the value of the Jacobian. However, based on the work of Sinha and Richards [32], it can be argued that, in the case of two modes—a barotropic mode (background current does not depend on depth) and the first baroclinic mode (nonlinear Rossby waves)—the interaction integral turns to zero, and we can assume that the dispersion relation in the WKB approximation would conserve its form (formula (17)).

DATA ANALYSIS

A region limited by borders 40°–60° S, 80°–120° E located in the Indian Ocean was selected for the analysis. Figure 1 clearly shows the ACC jet located in the band of latitudes between 45°–55° S manifested by the negative level values and maximum values of geostrophic currents. The isolines of the level and velocity values of the geostrophic currents have a predominantly zonal distribution. Isolines of kinetic energy also extend zonally. The values of the ADT level decrease towards the South Pole from 0.6 m at 40° S to –1.2 m at 60° S, and they change sign in the band 45°–48° S. The region in which the change of the ADT sign occurs is characterized by the maximum values of velocities and kinetic energy and, obviously, this fact determines the boundary of the waveguide where the energy of the waves and mesoscale eddies is trapped [12, 22]. One can see in Fig. 1b that the ACC flow is substantially nonuniform, and the geostrophic velocities averaged over a 25-year period vary over a wide range, reaching 50 cm/s; several jets with a pronounced eastern component can be distinguished. Overall, Fig. 1 demonstrates the capabilities of satellite altimetry for describing sea level variability and geostrophic circulation in a given basin.

Verification of the nonlinear theory in the ACC region was also carried out using the altimetry data: sea level anomalies were considered. Figure 2 shows the estimates of equivalent β^* calculated from the formula $\beta^* = \beta - U''_{yy}$. We note that, unlike parameter β, which is positive everywhere, the values of β^* in a given domain can be both positive and negative. It was also found that the values of β^*, varying from $-2 \cdot 10^{-11}$ to $4 \cdot 10^{-11}$ m^{-1} s^{-1}, generally exceed the values of β, which varies in this region from $1.1 \cdot 10^{-11}$ m^{-1} s^{-1} (at 60° S) to $1.7 \cdot 10^{-11}$ m^{-1} s^{-1} (at 40° S). This means that a significant contribution to formula (17) is made by the component related not to the β effect, but to the second term, the meridional change in the zonal flow shift, i.e., U''_{yy}.

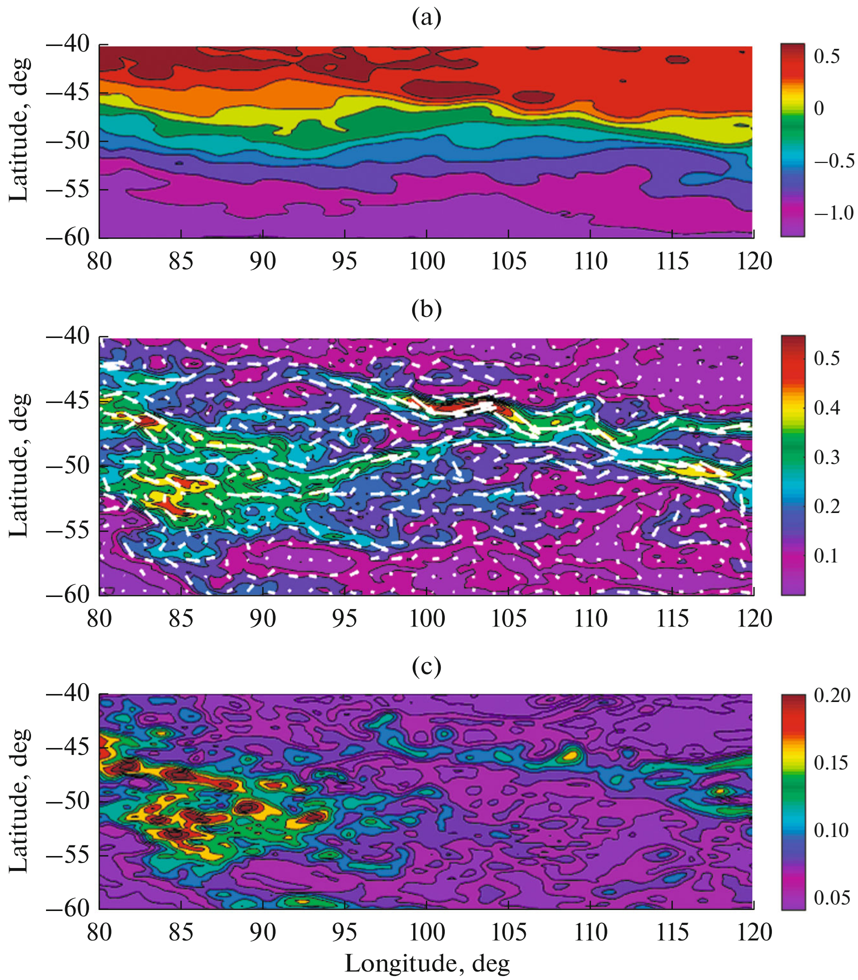

Fig. 1. Sea level (ADT) (m) (a), geostrophic currents (m/s) (b), and kinetic energy (m^2/s^2) (c). Averaging over the period 1993–2017.

Let us compare the theoretical phase velocities of Rossby waves, determined from the dispersion relations, with the "empirical" wave velocities calculated from the altimetry data. Empirical velocity estimates can be obtained using the Radon method, which was previously successfully tested in the Northwest Pacific [8]. In this method, the Rossby wave velocities are determined from the altimetry maps using the Radon transform [33] applied to the zonal spacetime sea level charts (Hovmöller diagrams). Figure 3 shows the empirical Rossby wave velocities calculated for this basin from the data in the period 1993–2016. It can be seen that the calculated velocities can be both negative and positive: the range of velocity variability is from −2 to 4 cm/s. Note that, in the northern part of the basin, the waves propagate only in the western direction; then as they approach the ACC zone, the western wave propagation is blocked and, starting from 44° S, the waves move not only to the west, but also to the east. Figure 3 identifies large regions with positive velocities reaching 4 cm/s in the core of the ACC around 46°–52° S. They correspond to the eastern direction of displacement of level inhomogeneities, which may mean that the Rossby waves are directed eastward here.

Figure 4 shows the spatial distribution of the baroclinic Rossby deformation radius R, and Fig. 5 shows the phase velocities, which are calculated using two approaches: from the classical formula for the phase velocity of the baroclinic Rossby waves in the longwave approximation: $c = -\beta R^2$, and from the formula (17): $c^* = -\beta^* R^2$. We note that the wave velocities calculated from the classical formula are negative everywhere; the velocities range from −2.0 to −0.2 cm/s; the Rossby waves propagate to the west. The spatial distribution of velocities in Fig. 5a has a zonal character, which is caused by the zonal changes of parameter β and the deformation radius (Fig. 4). At the same time, it can be seen in Fig. 5b that the velocities calculated from formula (17) can be both negative and positive, although there is also a tendency to a zonal distribution of velocities. The regions corresponding to the

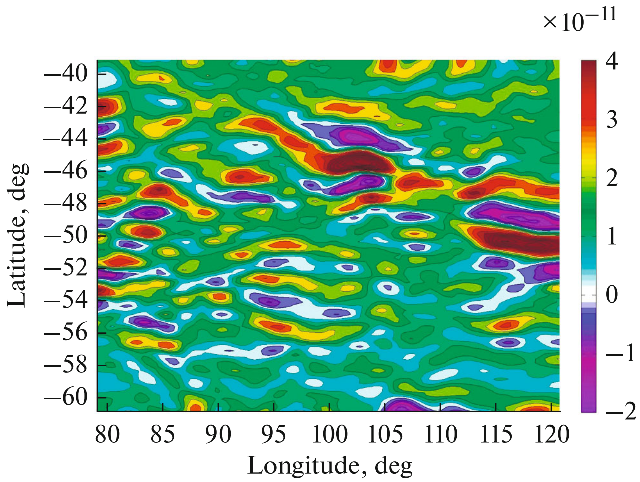

Fig. 2. $\beta^* = \beta - U''_{yy}$ ($m^{-1}\ s^{-1}$), calculations are based on the ADT (m) data, averaged over the period 1993–2016.

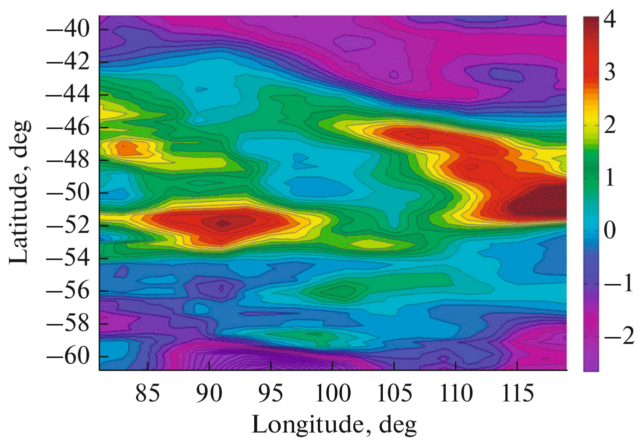

Fig. 3. Empirical phase velocities of Rossby waves (cm/s) calculated by the Radon method according to the altimetry data for the period 1993–2016.

propagation of the Rossby waves to the east with the maximum velocities equal to 1 cm/s are located in the core of the ACC; they are consistent with the arrangement of the regions with the positive values of the empirical velocities calculated using the Radon transform. From this we can conclude that the Rossby waves transform their energy so that they can change the direction of propagation and move not only to the west, but also to the east. These processes are due to the nonlinear interaction of Rossby waves with a current, in which the effect of relative vorticity can overlap the β effect. Using altimetry maps, we also compared the components of the meridional gradient of relative vorticity and found that the meridional changes in the shear of the zonal flow U''_{yy} exceed the values of V''_{xy} by one order of magnitude. This actually means that not only planetary, but also relative vorticity should be taken into account equally in the dispersion relation for the Rossby waves in a current.

Thus, we have shown that the nonlinear theory in the longwave approximation is applicable to describe the propagation of the Rossby waves (mesoscale eddies) along the flow. There are grounds to expect that formula (17) will also be valid for other jet streams, for example, the Kuroshio, although, of course, this statement requires verification.

It should be recognized that there is some difference between the empirical (Fig. 3) and theoretical (Fig. 5b) speeds of Rossby waves. In particular, the empirical wave speeds directed to the east reach positive values of 4 cm/s, while the maximum speeds from formula (17) are 1 cm/s. In addition, the extremes in Figs. 3 and 5b are not identical, although, in general, they are in good agreement with each other. Several explanations are possible. First, and most important, our nonlinear theory is considered in the longwave

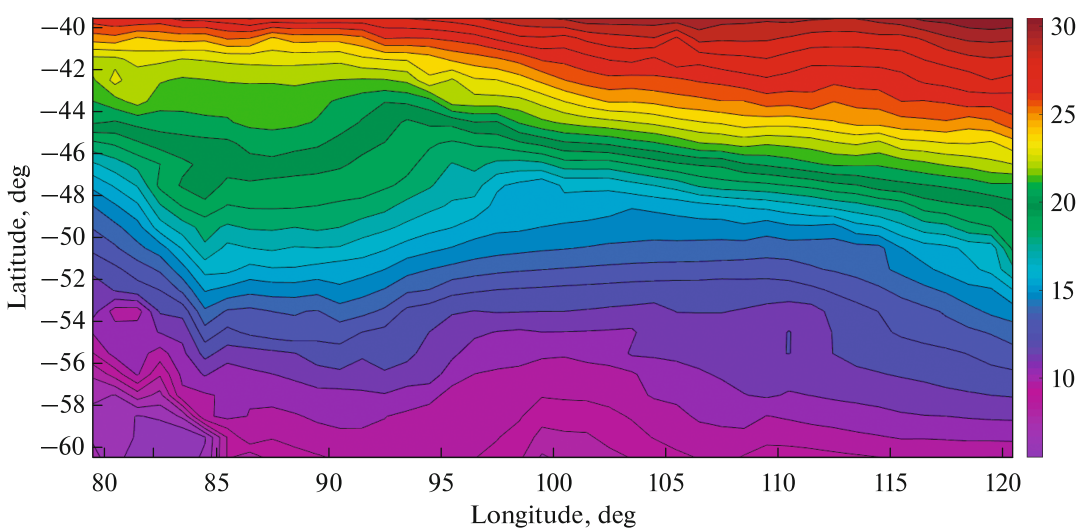

Fig. 4. Rossby deformation radius (km), built on a one-degree grid according to the resource http://www-po.coas.oregonstate.edu/research/po/research/rossby_radius/ (Chelton et al., 1998).

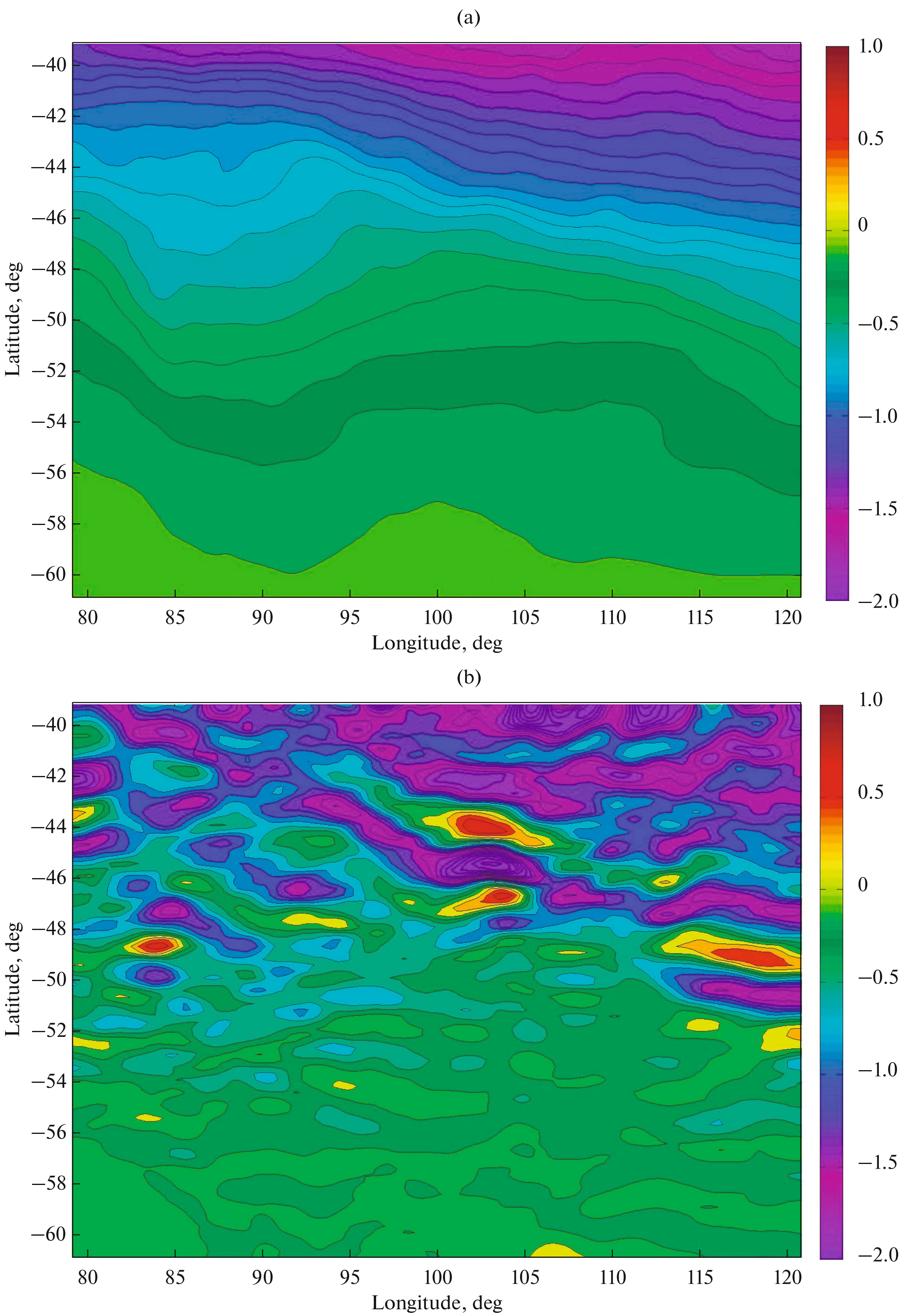

Fig. 5. Phase velocities calculated in the longwave approximation by the formulas: the classical dispersion relation $c = -\beta R^2$, (cm/s) (a) and according to the nonlinear theory $c^* = -\beta^* R^2$ (b).

approximation, while empirical velocities are calculated purely from observations using altimetry maps. Second, in this paper we consider only one nonlinear effect: the effect of shear flow, which we analyze using the example of the ACC. As stated above, this is actually a very complex structure of jets and meanders, which also depends on anemobaric conditions, on topography, etc. Third, there may be other factors that also affect the speed of the Rossby waves in the ocean. Nevertheless, we want to emphasize that the suggested new theory allowed us for the first time to describe the interaction of Rossby waves with a jet stream. This theory, sensitive to a significant difference in the speeds of Rossby waves and the current, allowed us for the first time to explain the eastward propagation of the Rossby waves trapped by the current, which we also observe when analyzing altimetry maps.

CONCLUSIONS

(1) A hypothesis was confirmed about the possibility of applying the model of the longwave asymptotics of nonlinear Rossby waves for an adequate description of the Rossby waves in a zonal stream.

(2) Within the framework of the nonlinear theory, it was shown that nonlinearity in the longwave approximation exactly compensates the Doppler shift. A new dispersion relation for the Rossby waves in a jet stream is derived.

(3) The suggested new approach is applied to the basin located in the ACC zone. This method has been verified based on the satellite altimetry data, and the structure of the Rossby waves in the ACC has been investigated.

(4) It has been shown that, in the ACC region, the values of β^* of the equivalent β effect also exceed the values of β by one order of magnitude. This means that the contribution of the component associated with the meridional change in the zonal flow shift U''_{yy} is greater than the influence of the β effect.

(5) A comparison of the velocities calculated from the altimetry data with the velocities determined from the nonlinear dispersion relation showed that it makes it possible to describe the eastward propagation of the Rossby waves trapped by the ACC current. The phase velocities of the Rossby waves calculated by the new dispersion relation depend on the meridional change in the zonal flow shear and can be both negative and positive in the ACC region. The maximum velocities corresponding to the motion of the waves to the east are 1 cm/s; they are characteristic of the ACC core. A tendency toward the zonal distribution of velocities exists in the spatial distribution of phase velocities due to the zonal changes in the Rossby deformation radius and parameter β.

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