On the problem of tsunami run-up to a flat shore

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Key points:

- A comparatively simple analytical solution to a nonlinear equation describing the tsunami run-up on a flat shore was found.

- The solution found describes a clear boundary of the front of the tsunami running ashore at a finite speed.

- Obtained solution indicates that rough shelf stops the tsunami wave by turbulence effect and it does not reach the coast

Abstract

When a tsunami wave comes from the ocean and propagates through the shelf, it is necessary to predict the maximum flooding of the coast, the height of the tsunami on the coast, the speed of the tsunami front through the coast, and other characteristics. A linear solution to this problem is unsatisfactory: it gives an infinite rate of coastal flooding, that is, the coast is flooded instantly and without a frontal boundary. In this study, we propose a new solution in nonlinear theory to calculate these tsunami characteristics. The obtained formulas show that the tsunami wave can be stopped on the shelf when approaching the shore. For this, it is necessary to artificially raise several tens of bottom protrusions to the level of calm water. Thus, the obtained solution allows to develop a physical-technical strategy to saving human lives and preventing material damage.

Plain Language Summary

The problem of reducing the impact force of tsunami, and consequently the reduction in the number of human casualties and the decrease of the level of destruction, is very significant. However, in order to understand the interaction of the tsunami with the shelf zone and the coastline, a convenient applied physical-geodynamic model of this phenomenon must be created. We found that the linear model is completely unsuitable for describing this complex natural phenomenon. On the basis of the many years of research experience in this area, we have found a nonlinear, relatively simple, but effective model of tsunami behavior near the coastline. Based on this model, a proposed solution allows to stop (or considerably weaken) the effect of the impending tsunami wave.

Introduction

Tsunami are long gravitational waves in the ocean occurring as a result of a short-term change in its volume, that is, due to large-scale disturbances in the ocean surface, its shores, or the bottom (Arsen'yev et al., 1998; Levin and Nosov, 2016; Rabinovich, 2020). Waves with a length λ exceeding the depth of the ocean H are called long waves ($\lambda > H$). Therefore, tsunami cover the entire ocean's thickness (in the concrete region) and can spread over transoceanic distances, that is, they are a planetary phenomenon like astronomical tides. Typical tsunami wave periods are from 1 minute to several hours, and characteristic wavelengths are from 1 km to 100 km. Therefore, when approaching the shelf, tsunami waves can nonlinearly interact with the shallow components of the ocean tide, which can weaken or strengthen the tsunami wave (Arsen'yev et al., 1993).

The tsunami phenomenon is a natural disaster, which has been intensively studied since the second half of the 20th century. Modern tsunami studies can be tentatively divided into three groups.

First, tsunami sources in the oceans and seas are being studied (e.g., Beisel et al., 2009; Wendt et al., 2009; Allgeyer and Cummins, 2014; Lay et al., 2016). Here, the waves are often calculated using the linear theory of potential, non-eddy motions of an ideal frictionless fluid under the influence of gravity field (Levin and Nosov, 2016). Such models are called non-hydrostatic, since they do not use shallow water equations and the hydrostatic law, which are valid for the long waves. In other models, tsunami waves are considered as long waves already at the source of excitement, therefore they are called as hydrostatic models (Garagash and Lobkovsky, 2006; Lobkovsky et al., 2019).

The second group of investigators studies the propagation of tsunami waves in the ocean (e.g., Beisel et al., 2009; Allgeyer, Cummins, 2014; Lay et al., 2016; Levin and Nosov, 2016; Wang et al., 2017). Here the waves are considered as long ones, the hydrostatic models of the theory of shallow water are used, and the process itself substantially depends on the depth of the ocean (Pelinovsky, 1996).

In the third group of works, the process of tsunami propagation through the continental shelf and coastal shallows is studied, including the process of transformation and destruction of waves upon running out to the land (e.g., Carrier and Greenspan, 1958; Arsen'yev, 1991; Arsen'yev et al., 1993; Didenkulova and Pelinovsky, 2000; Choi et al., 2006; Namekar et al., 2009; Satake et al., 2013; Montoya and Lynett, 2018).

This work belongs to the third group of studies. They are the most difficult, since is based on solving nonlinear equations. When a tsunami enters shallow water, nonlinear accelerations become significant, the wave height increases, bottom friction intensifies, and the motion becomes very turbulent. On the other hand, the stage of tsunami landfall is the most destructive, and its study is most important from both from scientific and practical points of view.

The problem of tsunami approaching the shore is often solved at present with the help of the Carrier-Greenspan transformation (Carrier and Greenspan, 1958). It allows to reduce the system of nonlinear equations of hydrodynamics to a linear wave equation with respect to the wave function for a given slope of the coast α. In this paper, we solve the problem of the tsunami wave run-up over a flat plain, considering the coastal slope absent ($\alpha = 0$). The area of flooding and the range of tsunami propagation inside into the land are in this case maximal. Thus, the found solutions to the problem, are of interest for numerous experts engaged in building construction, environment and safety in the coastal zone of the oceans and seas (Arsen'yev et al., 1998; Satake et al., 2013).

Statement of problem

We choose the origin of coordinates at the sea edge of the shelf $x = 0$. The x axis is directed along the wave propagation direction perpendicular to the coast, the y axis is perpendicular to the x axis (left), the z axis is down vertically (Figures 1 and 2). The letter M denotes the width of the shelf. Let us will select the level $z = 0$ at the surface of calm water, the letter ζ denotes the wave disturbance of the sea surface, and positive value ζ is counted down from the unperturbed level of $z = 0$ (Figure 2). The letter H and r denote the average depth of the shelf and the height of the protrusions of the roughness at the bottom, respectively. Thus, the total depth of the shelf is value of $H - r$.

We will use the equations of shallow water theory. They are obtained from the equations of geophysical hydrodynamics by integration along the z axis in the range from $z = \zeta$ to $z = H - r$ (Arsen'yev, 1991; Røed, 2014). Assuming that there are no changes along the y axis $\left(\frac{\partial}{\partial y} = 0\right)$, we write the initial equations in the form

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = g\frac{\partial \varsigma}{\partial x} - \frac{1}{\rho}\frac{\partial p^a}{\partial x} - \frac{\partial R_x^Z}{\partial z} \ . \tag{2}$$

Here u is the component of the flow velocity in the wave along the x axis, w is the velocity component along the z axis, p^a is the atmospheric pressure at the water surface, g is the gravity acceleration, R_x^z is the vertical component of the turbulent Reynolds stresses (Reynolds, 1894), and ρ is the density of water.

Estimates show that when a wave comes out in shallow water, the turbulent friction $\frac{\partial R_x^z}{\partial z}$ is two to three orders of magnitude greater than the nonlinear accelerations and the non-stationary term. Therefore, the equation (2) can be written as

$$g\frac{\partial \varsigma}{\partial x} = \frac{\partial R_x^z}{\partial z} + \frac{1}{\rho}\frac{\partial p^a}{\partial x} \quad . \tag{3}$$

It is necessary to add vertical boundary conditions to equations (1) and (3)

$$z = \varsigma, \qquad w = \frac{\partial \varsigma}{\partial t} + u\frac{\partial \varsigma}{\partial x}\ , \quad R_x^z = R_x^0, \tag{4}$$

$$z = H - r, \qquad u = w = 0\ , R_x^z = R_x^H. \tag{5}$$

Integrating equations (1) and (3) along the vertical axis from $z = \zeta$ to $z = H - r$ (it is the real depth, taking into account the protrusions of the roughness at the bottom), we obtain

$$\frac{\partial \varsigma}{\partial t} = \frac{\partial S}{\partial x}; \quad g\left(H - r - \varsigma\right)\frac{\partial \varsigma}{\partial x} - \left(\frac{H - r - \zeta}{\rho}\right)\frac{\partial p^a}{\partial x} = R_x^H - R_x^0\ , \tag{6}$$

and when integrating, we took into account the boundary conditions (4) and (5).

In equation (6), R_x^0 is the turbulent stress on the surface of the water caused by the action of the wind. This stress, as well as the atmospheric pressure gradient $\partial \rho^a / \partial x$, should be taken into account only when studying the processes of occurrence of storm surges and meteorological tsunamis (Arsen'yev and Shelkovnikov, 2010; Rabinovich, 2020). In our case, when studying the tsunami wave approach to the shore, these terms can be neglected. We associate the water turbulent friction on the bottom R_x^H with the total flow S by a linear law

$$R_x^H = \omega_T\, S\ , \ \omega_T = \frac{3A}{(H - r)^2}, \quad S = \int_{\varsigma}^{H - r} u\, dz\ . \tag{7}$$

Here ω_T is the friction frequency, and A is the shear vertical turbulent viscosity coefficient (Arsen'yev and Shelkovnikov, 2010).

Thus, equations (6) can be written as

$$\frac{\partial \varsigma}{\partial t} = \frac{\partial S}{\partial x}; \quad g\left(H - r - \varsigma\right)\frac{\partial \varsigma}{\partial x} = \omega_T \, S. \tag{8}$$

Two equations (8) can easily be reduced to one nonlinear equation of parabolic type with respect to the level ζ

$$\frac{\partial \varsigma}{\partial t} = \frac{\partial}{\partial x}\left[K(\varsigma)\, \frac{\partial \varsigma}{\partial x}\right], \tag{9}$$

in which the wave diffusion coefficient

$$K\left(\varsigma\right) = \frac{g\left(H - r\right)}{\omega_T} - \left(\frac{g}{\omega_T}\right)\varsigma \tag{10}$$

depends on an unknown quantity ζ.

Similar equations were studied in static physics (Boltzmann, 2011), in the theory of filtration (Boussinesq, 1904; Polubarinova-Kochina, 1971; Barenblatt, 1996), in the theory of atomic explosions (Zeldovich and Companaetz, 1950; Tikhonov and Samarskiy, 1963), in biomedical engineering (Kardashov et al., 1999; 2000), and in the theory of tornadoes (Arsen'yev et al., 2010). To solve them, numerical methods (Tikhonov and Samarskiy, 1963) and approximate analytical methods (Zeldovich and Companaetz, 1950; Polubarinova-Kochina, 1971; Barenblatt, 1996) have been developed. In this paper, we propose an elegant automodel solution to the problem, which describes the phenomenon under study with sufficient for practice accuracy.

Task solution

We first consider the simple case of a deep shelf, when $H - r >> \zeta$. Then equation (9) can be written as

$$\frac{\partial \varsigma}{\partial t} = K_L \, \frac{\partial^2 \varsigma}{\partial x^2}. \tag{11}$$

This is a classical parabolic equation of the type of the diffusion equation (or heat conduction) (Eppelbaum et al., 2014), which describes the process of tsunami wave dissipation on the shelf. It has the character of turbulent spreading with a diffusion coefficient

$$K_L = \frac{g\,H\,(1-n)}{\omega_T} = \frac{g\,H^3(1-n)^3}{3A}\;. \qquad (12)$$

Here $n = r/H$ is the relative roughness of the ocean bottom. This process can be understood by solving equation (11) with corresponding initial (13) and boundary conditions (14):

$$\text{by } t \le 0, \quad \zeta = 0 \quad \text{for all } x, \qquad (13)$$

$$\text{by } t > 0, \quad \zeta = \zeta_0\,; \ \text{by } x = 0, \ \zeta = 0 \quad \text{by } x \to \infty. \qquad (14)$$

As a result, we will be able to determine the horizontal emission of the tsunami wave to the shore, that is, the maximum range of tsunami propagation inland. We have

$$\varsigma = \varsigma_0\left[1 - \Phi\left(\frac{x}{2\sqrt{K_L\,t}}\right)\right], \qquad (15)$$

where

$$\Phi\left(\frac{x}{2\sqrt{K_L\,t}}\right) = \frac{2}{\sqrt{\pi}}\int_0^{\mu} \exp\left(-\eta^2\right) d\eta \qquad (16)$$

is the probability integral in which the upper limit $\mu = \frac{x}{\left[2(K_l t)^{1/2}\right]}$.

The thickness of the coastal strip flooded by the tsunami wave, i.e., a surge δ, can be found from the condition of a sufficiently noticeable decrease in the level ζ when moving away from the beginning $x = 0$

$$\varsigma = \varsigma_0\, erfc\left(\frac{\delta}{2\sqrt{K_L\,t}}\right) = 0.01\varsigma_0\,, \qquad (17)$$

where

$$erfc\left(\frac{x}{2\sqrt{K_L\,t}}\right) = 1 - \Phi\left(\frac{x}{2\sqrt{K_L\,t}}\right) \qquad (18)$$

is the additional probability integral.

The numerical value 0.01 is reached by the *erfc* function when the value of its argument $x\left[4K_L t\right]^{-1/2}$ is equal to two. Hence

$$\delta = 4\sqrt{K_L\,t} = 4\sqrt{\frac{g\,H\,(1-n)\,t}{\omega_T}}, \qquad (19)$$

or

$$\delta = 4\sqrt{\frac{g H^3 (1-n)^3 t}{3A}}. \tag{20}$$

It follows from equation (20) that the width of the flood zone δ does not depend on the amplitude of the tsunami wave ζ_0 falling to the shelf zone, does not depend on the width of the shelf *M*, but very strongly depends on the depth of the shelf *H*, the relative roughness $n = \frac{r}{H}$ and time *t* of tsunami action. Process of turbulence destroys the tsunami wave, therefore, with an increase in the shear turbulent viscosity coefficient *A*, the width of the flood zone δ decreases.

For A = 10 m^2/s, n = 0 (smooth bottom) and H = 10 m, from formula (20) follows that for t = 1 hour, δ = 4300 m. With a shelf width of M = 2000 m, the coast will be flooded by 2300 m. However, with a very rough bottom (reefs, rocky ledges at the bottom) when n = 0.5, we have (for the same depth, time and turbulent viscosity) from formula (20) δ = 1500 m, i.e. a wave the tsunami completely attenuates on the shelf with a width of M = 2000 m. We see that the tsunami attack can be stopped by creating flood barriers or berms on the shelf with a height of r = H. In this case, n = 1.1 - n = 0 and from formula (12) follows that $K_L = 0$. Equation (16) gives

$$\frac{2}{\sqrt{\pi}} \int_0^{\infty} \exp\left(-\eta^2\right) d\eta = 1 \tag{21}$$

and from solution of equation (15) we get $\zeta = 0$. Thus, the tsunami run-up stops on the shelf, and the coast remains dry (intact).

Note that the obtained solution is approximate and has two fundamental disadvantages. First, the width of the flood zone, strictly speaking, is infinite. And we cut it off artificially, using condition (17). Secondly, water spreads through the shelf and shore with infinite speed, which is unrealistic one. These shortcomings belong to any solution of a degenerate linear parabolic equation (11). However, as we will see now, they are absent in the solution of the nonlinear equation (9).

Let us introduce the length scale $h = H - r$, time scale $T = h^2/A$, dimensionless coordinate $\varphi = x/h$ and dimensionless time $\tau = t/T$. Then the dimensionless diffusion coefficient

$$\Lambda = \frac{K}{A} = \frac{g h^3 (1-e)}{3A^2} = G\vartheta, \tag{22}$$

where $e = \zeta/h$ is the dimensionless level disturbance, $G = \dfrac{gh^3}{3A^2}$ is some parameter (the authors of the paper suggest to call it as 'Galileo's number'), and $\theta = 1 - e$ is the relative water surface level.

Then equation (9) takes the form

$$\frac{\partial \vartheta}{\partial \varphi} = G\,\frac{\partial}{\partial \varphi}\left(\vartheta \frac{\partial \vartheta}{\partial \varphi}\right). \tag{23}$$

Its solution

$$\vartheta(\varphi,\tau) = \vartheta_0\,\tau\left(1 - \frac{\varphi}{c\,\tau}\right) \quad by \quad \varphi < c\,\tau, \tag{24}$$

$$\vartheta(\varphi,\tau) = 0 \quad by \quad \varphi \ge c\,\tau. \tag{25}$$

It is easy to verify that it satisfies not only equation (23), but the boundary condition at the beginning of coordinates $x = 0$, $\varphi = 0$ and the initial condition $\tau = t = 0$:

$$\vartheta\,(0,\tau) = \vartheta_0\,\tau, \quad \vartheta(\varphi,0) = 0. \tag{26}$$

Here θ_0 is the initial constant value. For example, for $\theta_0 = 1$, we have $\zeta_0 = 0$, i.e., there is no initial perturbation of the water surface level.

Indeed, substituting the solution (24) into equation (23), we obtain $c = (G\,\theta_0)^{1/2}$ and

$$c = \frac{h}{A}\sqrt{\frac{g\,(h - \varsigma_0)}{3}}. \tag{27}$$

The coordinates of the moving point x^* of the water edge, that is the nose of the tsunami wave running onto the shore (where $\theta = 1, \zeta = 0$), is determined from the equation

$$1 = \vartheta_0\,\tau\left(1 - \frac{\varphi^*}{c\,\tau}\right), \tag{28}$$

which is equivalent to the equation

$$\varphi^* = c\,\tau - \frac{c}{\vartheta_0}. \tag{29}$$

From this follows that $c = \dfrac{d\varphi^*}{d\tau} = \dfrac{T}{h}\dfrac{dx^*}{dt}$, or in dimensional form

$$V = \frac{dx^*}{dt} = \left(\frac{h}{T}\right)c = \sqrt{\frac{g\left(H - r - \varsigma_0\right)}{3}}. \qquad (30)$$

Tsunami nose coordinate (water edge) $x^* = \delta_n$ moves according to the law

$$x^* = \left(t - \frac{h^2}{A\vartheta_0}\right)\sqrt{\frac{g\left(H - r - \varsigma_0\right)}{3}}. \qquad (31)$$

Discussion

Solutions (24) - (30) describe simple, but actual physical-geodynamical model of tsunami wave running onto a coastal plain with a finite velocity of (30). It differs from the Lagrange velocity $(gH)^{1/2}$ of long waves, since roughness r, initial perturbation of the water level ζ_0 and turbulent friction are taken into account here. It can be seen from formula (27) that a tsunami wave with ζ_0 = 0 can be eliminated by creating roughness protrusions with a height of $r = H$, $h = 0$ at the bottom of the shelf. However, in contrast to the linear case, the tsunami wave is not just scattered over the shelf, but stops (or greatly weaken) because equation (30) indicates that its speed V vanishes.

Figure 3 shows the dependence of the total depth $D = H - z_0 - \zeta$ on the distance x for three time instants. Let us we stand on the shore of a beach with a width of M = 300 m near the water edge at a point x = 300 m from the beginning of coordinates, which is located on the sea edge of this beach (Fig. 1). Then the wave will begin to cover us, starting at the time t = 166 s, after the arrival of the wave from the origin x = 0. At time t = 387 s, the wave will lift us to a height of 2.21 m, and at time t = 664 s – to a height of 5 m. If after that, the flow of water to the origin ceases, $\theta(x = 0; \tau > 664\text{s}) = 1$, then the water that flooded the coastal plain on it will remain until the evaporation and infiltration into the soil will drain this coast.

The calculations shown in Fig. 3 are done for θ_0 = 1, ζ_0 = 0, depth h = 1 m and velocity V = 1.8 m/s. Analyzing Fig. 3, we see that the region covered by the tsunami $\partial_n = x^*$ is finite and moves at a speed V described in equation (30). Let us compare the size of the flood zone δ according to linear δ and nonlinear theory δ_n, setting H = 10 m, θ_0 = 1, ζ_0 = 0 and r = 0 (smooth bottom). According to the aforementioned nonlinear (more exact) theory, V = 5.71 m/s, and for 1 hour tsunami will flood an area of size δ_n = 20,500

m. The linear theory presented in equation (20) gives for A = 10 m^2/s the size of δ = 4,300 m, that is 4.7 times smaller.

For a very rough bottom, when H = 10 m, r = 5 m, h = 5 m, n = 0.5, we have V = 4 m/s and the nonlinear theory gives δ_n = 14,500 m. The linear theory at A = 10 m^2/s gives in this case δ = 1,530 m, that is 9.4 times less. As you can see, the linear approximation gives great errors. The fact is that the diffusion coefficient K in the equation (9) stands near the highest derivative. Therefore, the solutions of this equation substantially depend on the value of $K(\zeta)$.

The solution of equation (24) also makes it easy to reconstruct the dependence of the depth D and the average flow velocity $U = S/h$ on time t at various fixed distances x from the source. Corresponding nomograms and graphs can be used for engineering assessments in the construction of structures that will protect especially important objects (for example, nuclear and thermal power plants, chemical plants, airfields and others (Arsen'yev et al., 1998)) located nearly the shores of the seas and oceans from the tsunami phenomenon.

Conclusions

Let us state the main results obtained in this paper. Based on the nonlinear theory of shallow water, taking into account turbulent friction on a rough bottom, the theory of tsunami roll-up to a flat shore is constructed. Exact solutions of linearized and nonlinear equations are found. It is shown that the use of solutions of linearized equations leads to large errors. The obtained formulas make it easy to calculate the advancement of the water front inland, the height of flooding of the shelf and shore at a given point, the tsunami wave propagation range, the average current velocity in the wave, and other characteristics necessary for engineering calculations. It was established that the speed of the tsunami wave can be turned to zero, that is, the movement of the tsunami wave can be stopped when approaching the coast, on the shelf. To realize this, it is necessary to increase the height of the roughness protrusions (possibly using artificial adjustable structures) on the bottom of the shelf r to the level of undisturbed depth of the shelf H. The strong turbulent friction about the bottom that occurs destroys the tsunami wave on the shelf and the tsunami wave does not reach the shore.

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References

Allgeyer, S., and P. Cummins (2014), Numerical tsunami simulation including elastic lading and seas water density stratification, *Geophysical Research Letters*, *41*, 2308-2375, https://doi.org/10.1002/2014GL059348.

Arsen'yev, S.A. (1991), On the nonlinear equations of long sea waves, *Water Resources*, No.1, 29-35 (in Russian).

Arsen'yev, S.A., V.A. Babkin, A.Yu. Gubar, and V.N. Nikolaevskiy (2010), *Theory of Mesoscale Turbulence. Eddies of the Atmosphere and the Ocean. Regular and Chaotic Dynamics*, Inst. of Computer Sciences, Moscow-Izhevsk, 2010 (in Russian).

Arsen'yev, S.A., L.N. Rykunov, and N.K. Shelkovnikov (1993), Nonlinear interactions of tsunami and tides in an ocean, *Doklady of Russian Acad. of Sci*., *331*, No. 6, 732-734.

Arsen'yev, S.A., and N.K. Shelkovnikov (2010), Storm Surges as Dissipative Solitons. *Moscow Univ. Physics Bull*., *68*, No. 6, 483-489.

Arsen'yev, S.A., O.A. Zhivogina, S.V. Seleverstov, and N.K. Shelkovnikov (1998), Power influence of tsunami waves on the nuclear station, *Atomic Energy*, *85*, No. 1, 69-72 (in Russian).

Barenblatt, G.I. (1996), *Scaling, Self-Similarity, and Intermediate Asymptotic*, Cambridge Univ. Press, Cambridge.

Beisel, S., L. Chubarov, I. Didenkulova, E. Kil, A. Levin, E. Pelinovsky, Y. Shokin, and M. Sladkevich (2009), The 1956 Greek tsunami recorded at Yafo, Israel, and its numerical modeling. *Journal of Geophysical Research*, *114*, CO 9092, 1-18, https://doi.org/10.1029/2008JC005262.

Boltzmann, L. (2011), *Lectures on Gas Theory*, Dover Publ., Inc. NY, USA.

Boussinesq, J. (1904), Recherches theoriques sur L'ecoulement des nappes d'ean infiltrees dans le sol, *Jour. de math. pures et appl*., *Ser. 5, t. X*, fasc.1.

Carrier, G. F., and Greenspan H. P. (1958), Water waves of finite amplitude on a sloping beach, *Journal of Fluid Mechanics*, *4*, 97-109.

Choi, B.H., S.I. Hong, and E. Pelinovsky (2006), Distribution of run up heights of the December 26, 2004 tsunami in the Indian Ocean, *Geophysical Research Letters*, *33*, L13601, 1-4, doi: 10.1029/2006GL025867.

Didenkulova, I., and E. Pelinovsky (2020), Tsunami run-up on a plane beach in a tidal environment, *Pure and Applied Geophysics*, *177*, 1583-1593, doi.org/10.1007/s00024-019-02332-y

Eppelbaum, L.V., I.M. Kutasov, and A.N. Pilchin (2014), *Applied Geothermics*, Springer, Heidelberg – N.Y.

Garagash, L.A. and L.I. Lobkovsky (2006), An analysis of the dynamic displacement process of the sea bottom due to a subduction zone earthquake, *Trans. of the 4th Int. FLAC Symposium on Numerical Modelling in Geomechanics* (H. Varona, Ed.), Paper 06-01. Itaca Consulting Groups. Inc. Minneapolis, USA, 1-5.

Kardashov, V.R., S. Einav, L.V. Eppelbaum, and A. Ismail-Zadeh (1999), Novel approach to investigation and control of nonlinear nonstationary processes: application to environments and biomedical engineering, *Scientific Israel*, *No.3*, 24-33.

Kardashov, V.R., L.V. Eppelbaum, and O.V. Vasilyev (2000), The role of nonlinear source terms in geophysics, *Geophysical Research Letters*, *27*, No.14, 2069-2072, https://doi.org/10.1029/1999GL011192

Lay, T., L. Li, and K.F. Cheung (2016), Modelling tsunami observations to evaluate a proposed late tsunami earthquake stage for the 16 September 2015 Illapel, Chile, Mw 8.3 earthquake, *Geophysical Research Letters*, *43*, 7902-7912, doi: 10.1029/2006GL025867.

Levin, B.W., and M.A. Nosov (2016), *Physics of Tsunami*, Springer, AG Switzerland.

Lobkovsky, L.I., I.A. Garagash, and R.Kh. Mazova (2019), Numerical simulation of tsunami waves generated by the underwater landslide for the Northern Coast of the Black Sea (Dzhubga Area), *Geophysical Jour. International*, *218*, 1298–1306, doi: 10.1093/gji/ggz221

Montoya, L., and P. Lynett (2018), Tsunami versus infragravity surge: Comparison of the physical character of extreme run up, *Geophysical Research Letters*, *45*, 12,982-12,990, doi: 10.1029/2018GL080594

Namekar, S., Y. Yamasaki, and K.C. Cheung (2009), Neural network for tsunami and run up forecast, *Geophysical Research Letters*, *36*, L08604, 1-6, doi:10.1029/2009GL037184

Pelinovsky, E.N. (1990), *Hydrodynamics of Tsunami Waves*, Inst. of Applied Physics of the Russ. Acad. of Sci., Nizhny Novgorod, Russia (in Russian).

Polubarinova-Kochina, P.Ya. (1971), *Theory of Ground Water Movement*, Nauka, Moscow (in Russian).

Rabinovich, A.B. (2020), Twenty-seven years of progress in the science of meteorological tsunami following the 1992 Daytona Beach events, *Pure and Applied Geophysics*, *177*, 1193-1230, doi.org/10.1007/s00024-019-02349-3

Reynolds, O. (1894), On the dynamical theory of incompressible viscous fluids and the determination of the criterion, *Philosoph. Trans. Royal Soc*., London, *186*, 123-161.

Røed, L.P. (2014), *Fundamentals of Atmospheres and Oceans on Computers*, University of Oslo, Institute of Geosciences.

Satake, K., Y. Fujii, T. Harada, and Y. Namegaya (2013), Time and space distribution of coseismic slip of the 2011 Tohoku earthquake as inferred from tsunami waveform data, *Bull. of Seismological Society of America*, *103*, No. 2B, 1165-1170, https://doi.org/10.1785/0120120122

Tikhonov, A.N., and A.A. Samarsky (1963), *Equations of Mathematical Physics*, Pergamon Press, Oxford.

Wang, Y., K. Satake, T. Maeda, and A.B. Gusman (2017), Green's function - based tsunami data assimilation: A fast data assimilation approach toward tsunami early warning, *Geophysical Research Letters*, *44*, 10,282-10,289, doi:10.1002/2017GL075307

Wendt, J., D.D. Oglesby, and E.L. Geist (2009), Tsunami and splay fault dynamics, *Geophysical Research Letters*, *36*, L15303, 1-5, doi: 10.1029/2009GL038295

Zeldovich, Ya.B., and A.S. Companaetz (1950), On the theory of heat propagation with temperature-depended thermal conductivity, *Coll. of papers dedicated to the 70th anniver. of Academician A.F. Ioffe*. USSR Acad. of Sci., Moscow, 61-71 (in Russian).

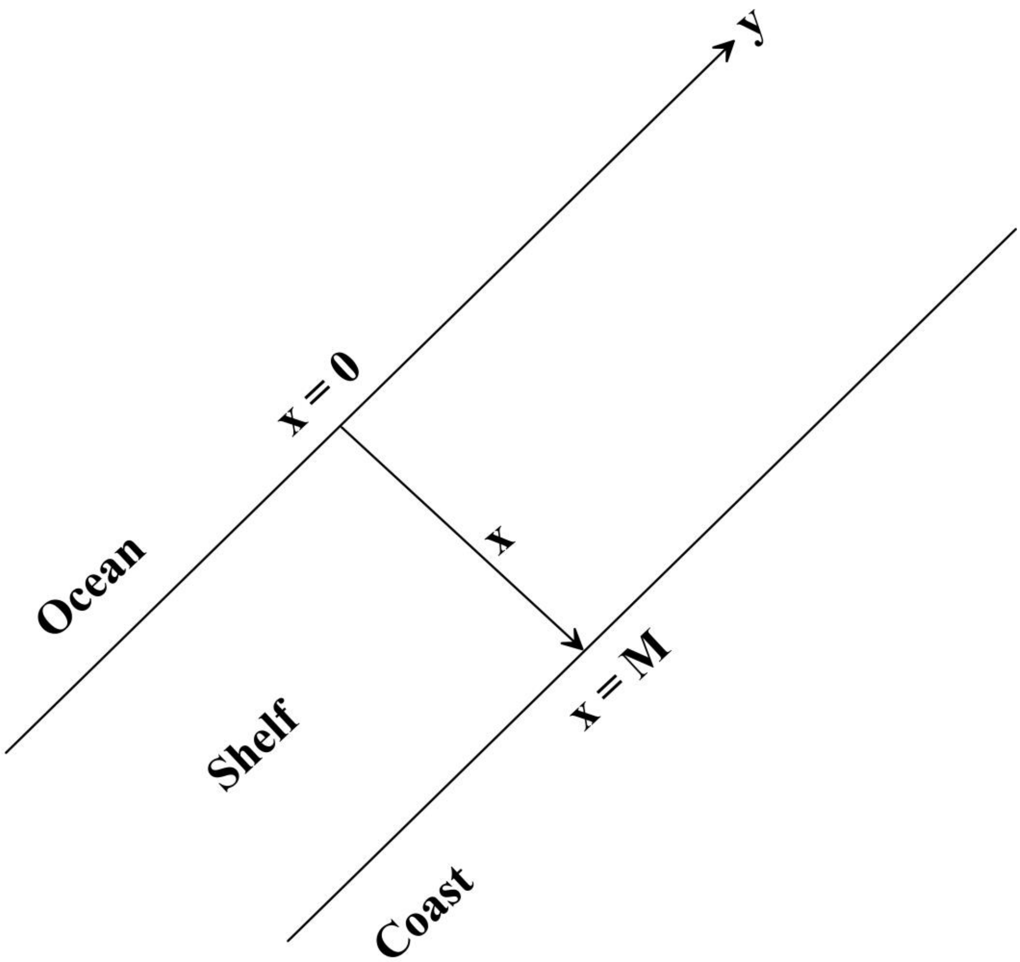

Figure 1. Horizontal coordinate axes and designations

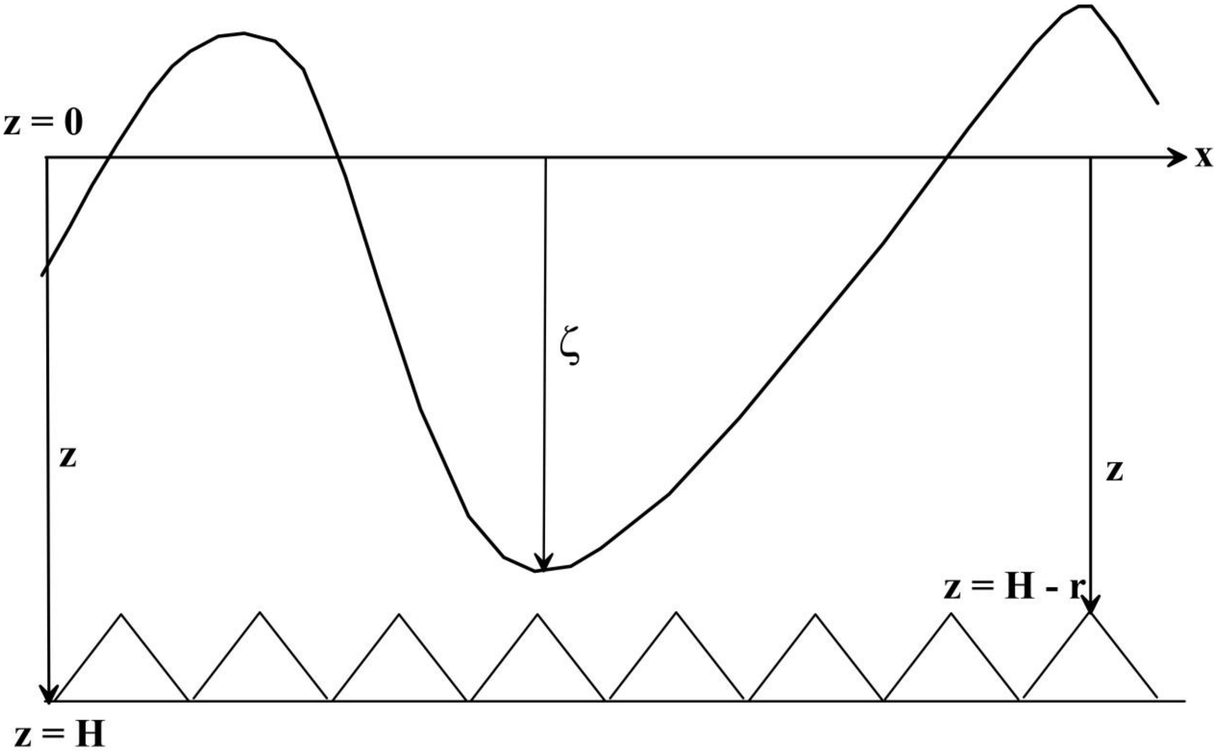

Figure 2. Vertical section of the water flow and corresponding designations

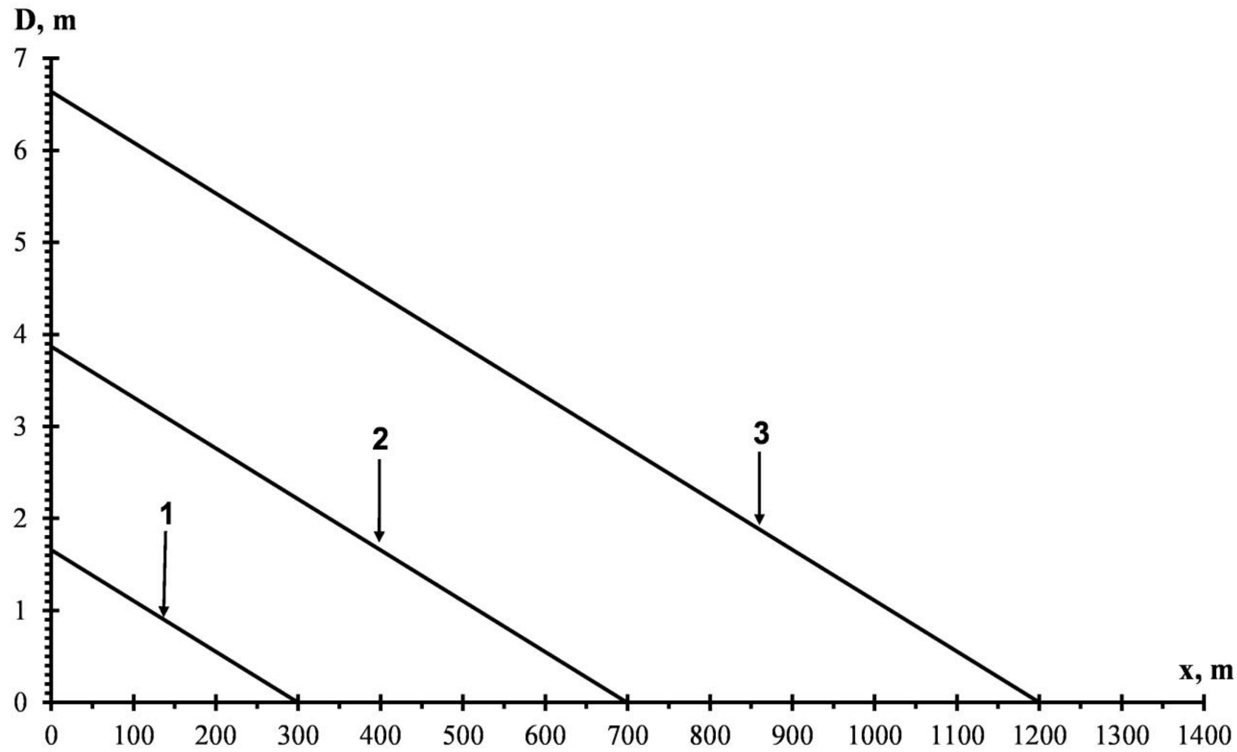

Figure 3. Dependence of the total depth of water flow D on the distance x at various time intervals. Graph $1-t$ = 166 s, water edge is located at x = 300 m. Graph $2-t$ = 387 s, water edge is located at x = 700 m. Graph $3-t$ = 664 s, water edge is located at x = 1200 m.